From Trees to Graphs: Understanding the Implications of Sharing for Rewriting

**Motivation**
- rewriting = transformation of objects based on rules
  - term rewriting [1,2] objects are terms = trees
  - from trees to graphs ~ term graph rewriting
  - sharing of equal subterms, avoids blow-up in size

- influences the potential rewrite steps with rule:

**Aim:** understand the implication of sharing on rewriting & on termination, i.e., the absence of infinite rewrite sequences

**Term Graphs**
- term graph rewriting with explicit sharing
- simulates term rewriting with linear size growth and polynomial overhead [3,4]
- but counter-intuitive, hence investigate with only

**Contribution**
- construct from Top and argument graph
  - contradiction to T is "bad"

**Proof Sketch:** (Kruskal’s tree theorem [7], minimal bad sequences [8])
- $\sqsubseteq$ is a well-quasi order if all infinite sequences are “good”
  - “good” means for some $i < j$: $A_i \sqsubseteq A_j$
- construct minimal “bad” sequence $T$
  - “bad” means for all $i < j$: $A_i \not\sqsubseteq A_j$

- take arguments of $T$

**Summary:** By moving from a tree to a graph representation, the termination behavior of rewriting changes. I re-proved that an order on the top of term graphs can be extended to an order on term graphs. This is the basis for constructing a termination order and enabling automated termination analysis.

**Background & Related Work**
- term graph rewriting with explicit sharing and unsharing
- simulates term rewriting with linear size growth and polynomial overhead [3,4]
- but counter-intuitive, hence investigate with only $\sqsubseteq$

Every term graph rewrite step can be simulated by n term rewrite steps.

Termination of term rewriting
- see infinite rewrite sequence with rule:

**Termination**
- $\Rightarrow$ termination of term graph rewriting [5]
- re-prove directly with Kruskal’s tree theorem
- insight: view arguments as one argument graph

**Vision**
- basis for a termination order on term graphs [6]
- basis for an automated termination analysis for term graph rewriting
- applications for term graph rewriting

**References**
5. Plump, D: Term Graph Rewriting (1999)
6. Plump, D: Simplification Orders for Term Graph Rewriting (1997)