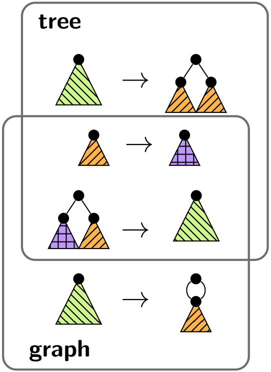
From Trees to Graphs: Kruskal's Tree Theorem & Termination

master's thesis @ University of Innsbruck, Austria

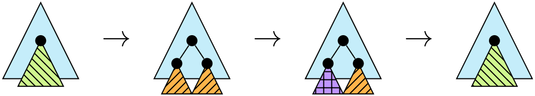
Maria A Schett
mail@maria-a-schett.net

June 23, 2016 @ OPLSS

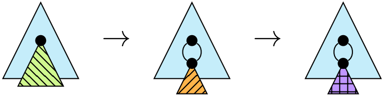
rules



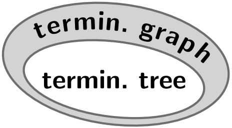
tree



graph



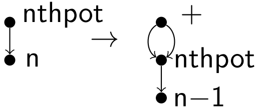
- termination: not ∞-many "$\rightarrow$"-steps



. . . motivational slide

- rewriting is Turing-complete model of computation
  analyzing "real-world" programming languages

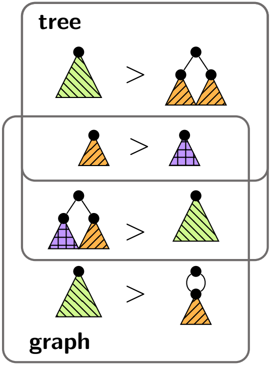
```
nthpot 0 = 1
nthpot n = x + x
  where x = nthpot (n−1)
```



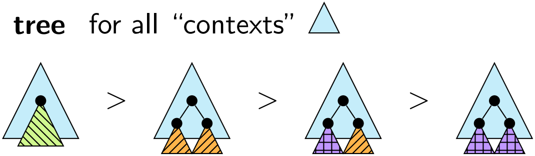
- tools: cf. `termCOMP`
  - ⋆ termination: $\mathsf{T_TT_2}$, AProVE, T2, WANDA, . . .
  - ⋆ complexity: TcT, AProVE, . . .
  - ⋆ C and Java: AProVE, Ultimate Büchi Automizer, COSTA, . . .

termination

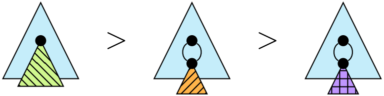
rules

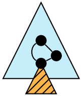


tree for all "contexts"



graph



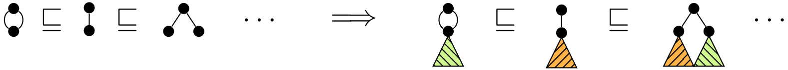


- because we distinguish and
- . . . but for all "contexts"

Kruskal's Tree Theorem & termination

Kruskal's Tree Theorem.

wqo $\sqsubseteq$ on   wqo $\sqsubseteq_{\text{emb}}$ on



all ∞ sequences are good, i.e. $\exists i < j$ s.t. $\triangle_i \sqsubseteq_{\text{emb}} \triangle_j$

Proof. following Nash-Williams' minimal bad sequence argument.

Termination. if $\sqsupset_{\text{emb}} \subseteq >$ then not ∞-many "$\rightarrow$"-steps

- assume ∞-many "$\rightarrow$"-steps: $\triangle_1 > \triangle_2 > \triangle_3 \quad \ldots$
- by Kruskal's Tree Theorem: $\triangle_i \sqsubseteq_{\text{emb}} \triangle_j$
- by $\sqsupset_{\text{emb}} \subseteq >$: $\triangle_i \leqslant \triangle_j$
- by assumption (and transitivity): $\triangle_i > \triangle_j$

wrap up . . .

References

D Plump.
Simplification Orders for Term Graph Rewriting
Proc. Math. Found. of Computer Science, LNCS vol. 1295, pp. 458–467, 1997.

G Moser, M A Schett.
Kruskal's Tree Theorem for Acyclic Term Graphs
Proc. 9th International Workshop on Computing with Terms and Graphs , 2016.

Thank you for your attention!

`mail@maria-a-schett.net`