From Trees to Graphs: Kruskal’s Tree Theorem & Termination

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rewriting

termination: not $\infty$-many “$\rightarrow$”-steps
rewriting is Turing-complete model of computation analyzing “real-world” programming languages

\[ \text{nthpot } 0 = 1 \]
\[ \text{nthpot } n = x + x \]
\[ \text{where } x = \text{nthpot} (n - 1) \]

tools: cf. termCOMP

- termination: \( T \upharpoonright T_2 \), AProVE, T2, WANDA, ...
- complexity: \( TcT \), AProVE, ...
- C and Java: AProVE, Ultimate Büchi Automizer, COSTA, ...
termination

rules

tree

graph

because we distinguish and

...but for all “contexts”
Kruskal’s Tree Theorem & termination

Kruskal’s Tree Theorem.

\[
\begin{align*}
\text{wqo} & \sqsubseteq \text{on} \\
\begin{array}{ccc}
\bullet & \subseteq & \bullet \\
\bullet & \subseteq & \ast \\
\ast & \subseteq & \ast \\
\end{array}
& \implies
\begin{array}{ccc}
\bullet & \subseteq & \bullet \\
\ast & \subseteq & \ast \\
\ast & \subseteq & \ast \\
\end{array}
\end{align*}
\]

all $\infty$ sequences are good, i.e. $\exists i < j$ s.t. $\triangle_i \sqsubseteq_{\text{emb}} \triangle_j$

Proof. following Nash-Williams’ minimal bad sequence argument.

Termination. if $\sqsubseteq_{\text{emb}} \subseteq >$ then not $\infty$-many “$\rightarrow$”-steps

- assume $\infty$-many “$\rightarrow$”-steps: $\triangle_1 > \triangle_2 > \triangle_3 \ldots$
- by Kruskal’s Tree Theorem: $\triangle_i \sqsubseteq_{\text{emb}} \triangle_j$
- by $\sqsubseteq_{\text{emb}} \subseteq >$: $\triangle_i \leq \triangle_j$
- by assumption (and transitivity): $\triangle_i > \triangle_j$
References

D Plump. Simplification Orders for Term Graph Rewriting

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Thank you for your attention!
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