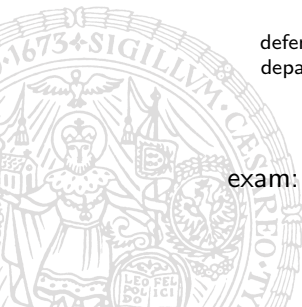# From Trees to Graphs: On the Influence of Collapsing on Rewriting and on Termination

Maria A Schett

defensio: master thesis advised by Georg Moser
department of CS @ UIBK   Oktober 25, 2016
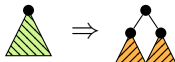
chair: Michael Felderer

exam: Georg Moser & Sebastiaan Joosten

Rewriting: from Trees to Graphs

- rewriting: replacing equals by equals with directed equations "$\Rightarrow$"

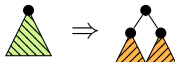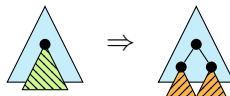**rules**   **tree $\approx$ term**

Rewriting: from Trees to Graphs

- rewriting: replacing equals by equals with directed equations "$\Rightarrow$"

rules   **tree $\approx$ term**

Rewriting: from Trees to Graphs

- rewriting: replacing equals by equals with directed equations "$\Rightarrow$"

rules

tree

tree $\approx$ term

term graph

term graph

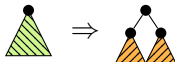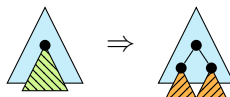## Rewriting: from Trees to Graphs

- rewriting: replacing equals by equals with directed equations "$\Rightarrow$"

rules

tree

tree $\approx$ term

term graph

term graph

Rewriting: from Trees to Graphs

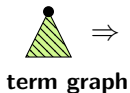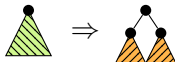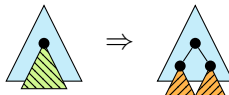- rewriting: replacing equals by equals with directed equations "$\Rightarrow$"

rules

tree $\approx$ term

term graph

Rewriting: from Trees to Graphs

- rewriting: replacing equals by equals with directed equations "⇒"

rules

tree

tree ≈ term

term graph

Rewriting: from Trees to Graphs

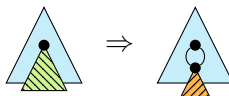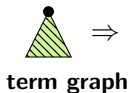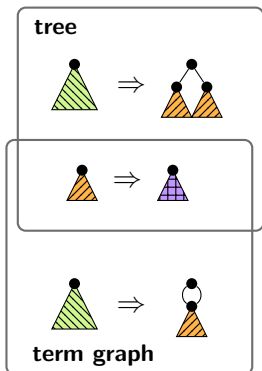- rewriting: replacing equals by equals with directed equations "$\Rightarrow$"

rules

tree $\approx$ term

term graph

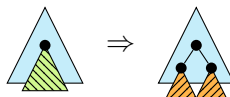## Rewriting: from Trees to Graphs. . .

- rewriting: replacing equals by equals with directed equations "⇒"

rules

tree

tree ≈ term

. . . changes the potential "⇒"-steps!

term graph
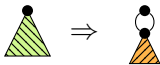
Agenda

① **Termination**

② **Collapsing**

③ **Literature**

① **Termination**

- not infinitely many "$\Rightarrow$"-steps
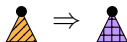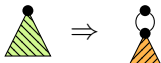
① **Termination**
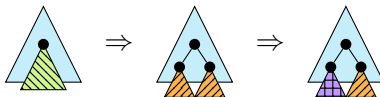
- not infinitely many "$\Rightarrow$"-steps

rules

tree $\approx$ term

term graph

① **Termination**

- not infinitely many "⇒"-steps
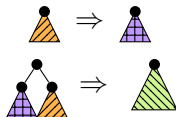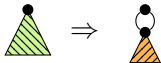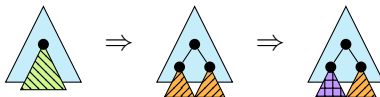
① **Termination**

- not infinitely many "⇒"-steps
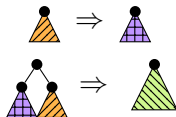
rules

tree ≈ term
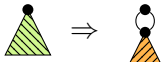
term graph

① **Termination**

termination graph

termination tree

• not infinitely many "⇒"-steps

rules
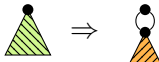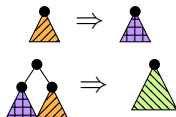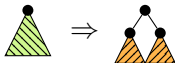
tree ≈ term
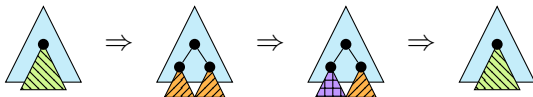
term graph

① **Termination**

- **Goal:** design method to show termination of term graph rewriting[1]

---

[1]inspired by

📄 D Plump. Simplification Orders for Term Graph Rewriting *Proc. Math. Found. of CS*, LNCS vol. 1295, pp. 458–467, 1997.

① **Termination**

- **Goal:** design method to show termination of term graph rewriting[1]
- **Method:** find order $>_{\text{lpo}}$ on term graphs

---

[1]inspired by

📄 D Plump. Simplification Orders for Term Graph Rewriting *Proc. Math. Found. of CS*, LNCS vol. 1295, pp. 458–467, 1997.

① **Termination**

- **Goal:** design method to show termination of term graph rewriting[1]
- **Method:** find order $>_{\mathsf{lpo}}$ on term graphs

Main ideas: given $\begin{matrix} f \\ ( ) \\ \bullet \end{matrix}$ vs $\begin{matrix} f \\ \bullet\ \bullet \end{matrix}$

---

[1]inspired by

📄 D Plump. Simplification Orders for Term Graph Rewriting *Proc. Math. Found. of CS*, LNCS vol. 1295, pp. 458–467, 1997.

① **Termination**

- **Goal:** design method to show termination of term graph rewriting[1]
- **Method:** find order $>_{lpo}$ on term graphs

Main ideas: given vs

"Top"

 vs

---

[1]inspired by

📄 D Plump. Simplification Orders for Term Graph Rewriting *Proc. Math. Found. of CS*, LNCS vol. 1295, pp. 458–467, 1997.

① Termination

- **Goal:** design method to show termination of term graph rewriting[1]
- **Method:** find order $>_{lpo}$ on term graphs

Main ideas: given vs

"Top" "Argument"

 vs   vs

---

[1]inspired by

📄 D Plump. Simplification Orders for Term Graph Rewriting *Proc. Math. Found. of CS*, LNCS vol. 1295, pp. 458–467, 1997.

① **Termination**

- **Goal:** design method to show termination of term graph rewriting[1]
- **Method:** find order $>_{\text{lpo}}$ on term graphs

Main ideas: given vs

"Top" vs

"Argument" vs

"Left-of" vs

---

[1]inspired by

📄 D Plump. Simplification Orders for Term Graph Rewriting *Proc. Math. Found. of CS*, LNCS vol. 1295, pp. 458–467, 1997.

① **Termination**

Procedure[2]

① define embedding relation $\sqsubseteq_{emb}$ wrt. Top, Argument, and Left-of

---

[2] G Moser, M A Schett. Kruskal's Tree Theorem for Acyclic Term Graphs *Proc. 9th TERMGRAPH*, 2016.

① **Termination**

Procedure[2]

① define embedding relation $\sqsubseteq_{emb}$ wrt. Top, Argument, and Left-of

② show **Kruskal's Tree Theorem for term graphs**

---

[2] G Moser, M A Schett. Kruskal's Tree Theorem for Acyclic Term Graphs *Proc. 9th TERMGRAPH*, 2016.

① **Termination**

Procedure[2]

① define embedding relation $\sqsubseteq_{emb}$ wrt. Top, Argument, and Left-of

② show **Kruskal's Tree Theorem for term graphs**

wqo $\sqsubseteq$ on Tops implies wqo $\sqsubseteq_{emb}$ on term graphs

proof by minimal bad sequence argument

---

[2] G Moser, M A Schett. Kruskal's Tree Theorem for Acyclic Term Graphs *Proc. 9th TERMGRAPH*, 2016.

① **Termination**

Procedure[2]

① define embedding relat for any infinite sequence hent, and Left-of

② show **Kruskal's Tree** $\exists\, i < j$ s.t. $\triangle_i \sqsubseteq_{\mathsf{emb}} \triangle_j$ **s**

wqo $\sqsubseteq$ on Tops implies wqo $\sqsubseteq_{\mathsf{emb}}$ on term graphs

$$\underset{\bullet}{\overset{f}{()}} \sqsubseteq \overset{g}{\underset{\bullet}{|}} \sqsubseteq \overset{f}{\underset{\bullet\ \bullet}{\wedge}} \sqsubseteq \ldots \text{ implies } \overset{f}{\underset{\triangle}{\bullet}} \sqsubseteq_{\mathsf{emb}} \overset{g}{\underset{\triangle}{\bullet}} \sqsubseteq_{\mathsf{emb}} \overset{f}{\underset{\triangle\ \triangle}{\bullet\ \bullet}} \sqsubseteq_{\mathsf{emb}} \ldots$$

proof by minimal bad sequence argument

---

[2] G Moser, M A Schett. Kruskal's Tree Theorem for Acyclic Term Graphs *Proc. 9th TERMGRAPH*, 2016.

① **Termination**

Procedure[2]

① define embedding relation $\sqsubseteq_{emb}$ wrt. Top, Argument, and Left-of

② show **Kruskal's Tree Theorem for term graphs**

wqo $\sqsubseteq$ on Tops implies wqo $\sqsubseteq_{emb}$ on term graphs

$$\overset{f}{\underset{\bullet}{()}} \sqsubseteq \overset{g}{\underset{\bullet}{|}} \sqsubseteq \overset{f}{\underset{\bullet\ \bullet}{/\backslash}} \sqsubseteq \ldots \text{ implies } \quad \overset{f}{\underset{\triangle}{\bullet}} \sqsubseteq_{emb} \overset{g}{\underset{\triangle}{\bullet}} \sqsubseteq_{emb} \overset{f}{\underset{\triangle\ \triangle}{\bullet\ \bullet}} \sqsubseteq_{emb} \ldots$$

proof by minimal bad sequence argument

③ if $\sqsubseteq_{emb} \subseteq <_{lpo}$
then not ∞-many "⇒"-steps thus **termination**

---

[2] G Moser, M A Schett. Kruskal's Tree Theorem for Acyclic Term Graphs *Proc. 9th TERMGRAPH*, 2016.

① **Termination**

Procedure[2]

① define embedding relation $\sqsubseteq_{emb}$ wrt. Top, Argument, and Left-of

② show **Kruskal's Tree Theorem for term graphs**

wqo $\sqsubseteq$ on Tops implies wqo $\sqsubseteq_{emb}$ on term graphs

proof by minimal bad sequence argument

③ if $\sqsubseteq_{emb} \subseteq <_{lpo}$
then not ∞-many "⇒"-steps thus **termination**

---

[2] G Moser, M A Schett. Kruskal's Tree Theorem for Acyclic Term Graphs *Proc. 9th TERMGRAPH*, 2016.

Agenda

② **Collapsing**

② **Collapsing**

- explicitly share equal sub-graphs "collapsing"

② **Collapsing**
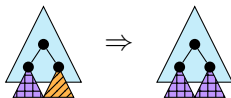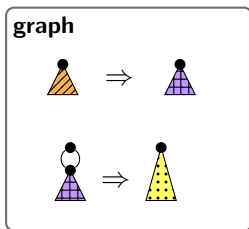
- explicitly share equal sub-graphs "collapsing"

rules

② **Collapsing**

- explicitly share equal sub-graphs "collapsing"
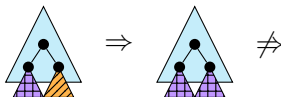
rules

② **Collapsing**

- explicitly share equal sub-graphs "collapsing"
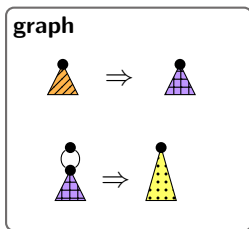
rules

② **Collapsing**

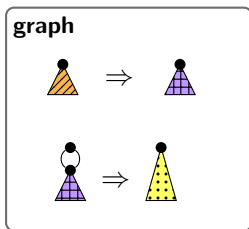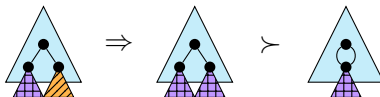- explicitly share equal sub-graphs "collapsing"
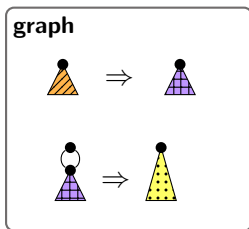
rules

② **Collapsing**

• explicitly share equal sub-graphs "collapsing"
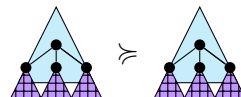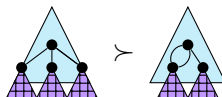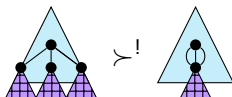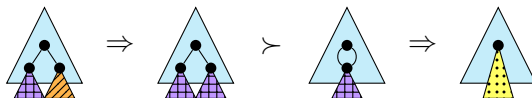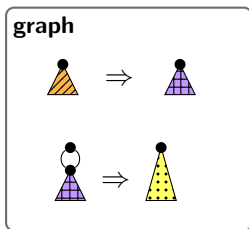
rules

② **Collapsing**

- explicitly share equal sub-graphs "coll

How to combine
$\Rightarrow$ & $\succcurlyeq$ or $\succ$ or $\succ^!$?

rules

② **Collapsing**

- combination through concatenation or union
- comparison wrt. **single steps** and **normal forms**

② Collapsing

- combination through concatenation or union
- comparison wrt. **single steps** and **normal forms**

$$\Rightarrow \cdot \not\succeq / \not\succeq \cdot \Rightarrow \quad \Rightarrow \cdot \succ^! / \succ^! \cdot \Rightarrow \quad \Rightarrow \cup \succ \quad \Rightarrow \cup \succ^!$$

				$\Rightarrow$
				$\Rightarrow \cdot \not\succeq / \not\succeq \cdot \Rightarrow$
				$\Rightarrow \cdot \succ^! / \succ^! \cdot \Rightarrow$
				$\Rightarrow \cup \succ$

② **Collapsing**

- combination through concatenation or union
- comparison wrt. **single steps** and **normal forms**

$$\Rightarrow \cdot \succsim / \succsim \cdot \Rightarrow \quad \Rightarrow \cdot \succ^! / \succ^! \cdot \Rightarrow \quad \Rightarrow \cup \succ \quad \Rightarrow \cup \succ^!$$

				$\Rightarrow$
		$\star$		$\Rightarrow \cdot \succsim / \succsim \cdot \Rightarrow$
				$\Rightarrow \cdot \succ^! / \succ^! \cdot \Rightarrow$
				$\Rightarrow \cup \succ$

② **Collapsing**

- combination through concatenation or union
- comparison wrt. **single steps** and **normal forms**

$\Rightarrow \cdot \succcurlyeq / \succcurlyeq \cdot \Rightarrow \quad \Rightarrow \cdot \succ^! / \succ^! \cdot \Rightarrow \quad \Rightarrow \cup \succ \quad \Rightarrow \cup \succ^!$

		$\star$	

$\Rightarrow$
$\Rightarrow \cdot \succcurlyeq / \succcurlyeq \cdot \Rightarrow$
$\Rightarrow \cdot \succ^! / \succ^! \cdot \Rightarrow$
$\Rightarrow \cup \succ$

$$(\Rightarrow \cup \succ)^{O(n^2)} = \succcurlyeq \cdot (\Rightarrow \cdot \succcurlyeq)^n$$

② Collapsing

- combination through concatenation or union
- comparison wrt. **single steps** and **normal forms**

$$\Rightarrow \cdot \succcurlyeq / \succcurlyeq \cdot \Rightarrow \quad \Rightarrow \cdot \succ^! / \succ^! \cdot \Rightarrow \quad \Rightarrow \cup \succ \quad \Rightarrow \cup \succ^!$$

				$\Rightarrow$
				$\Rightarrow \cdot \succcurlyeq / \succcurlyeq \cdot \Rightarrow$
			$\star$	$\Rightarrow \cdot \succ^! / \succ^! \cdot \Rightarrow$
				$\Rightarrow \cup \succ$

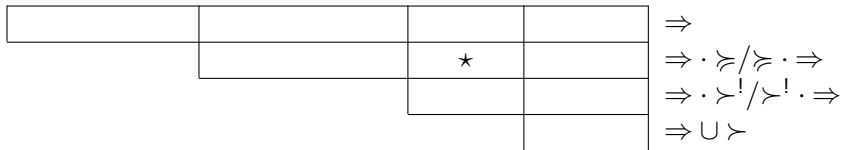$$(\Rightarrow \cup \succ^!)^{O(n^2)} \supsetneq \succ^! \cdot (\Rightarrow \cdot \succ^!)^n$$

② **Collapsing**

- combination through concatenation or union
- comparison wrt. **single steps** and **normal forms**

$$\Rightarrow \cdot \succsim / \succsim \cdot \Rightarrow \quad \Rightarrow \cdot \succ^! / \succ^! \cdot \Rightarrow \quad \Rightarrow \cup \succ \quad \Rightarrow \cup \succ^!$$

			$\star$

$\Rightarrow$
$\Rightarrow \cdot \succsim / \succsim \cdot \Rightarrow$
$\Rightarrow \cdot \succ^! / \succ^! \cdot \Rightarrow$
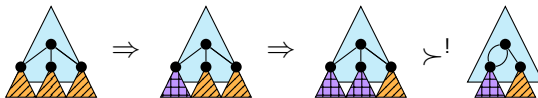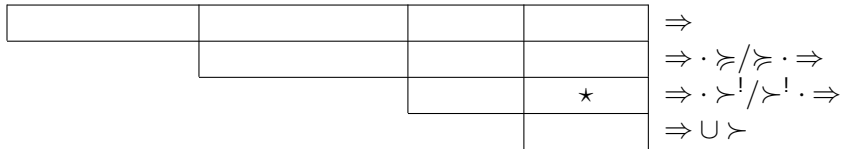$\Rightarrow \cup \succ$

$$(\Rightarrow \cup \succ^!)^{O(n^2)} \supsetneq \succ^! \cdot (\Rightarrow \cdot \succ^!)^n$$

② **Collapsing**

- combination through concatenation or union
- comparison wrt. **single steps** and **normal forms**

$$\Rightarrow \cdot \succsim/\succsim \cdot \Rightarrow \quad \Rightarrow \cdot \succ^!/\succ^! \cdot \Rightarrow \quad \Rightarrow \cup \succ \quad \Rightarrow \cup \succ^!$$

$\star$		$\star$	$\star$	$\Rightarrow$
	$\star$	$\star$	$\star$	$\Rightarrow \cdot \succsim/\succsim \cdot \Rightarrow$
		$\star$	$\star$	$\Rightarrow \cdot \succ^!/\succ^! \cdot \Rightarrow$
			$\star$	$\Rightarrow \cup \succ$

Agenda

③ **Literature**

③ Literature

Different Representations and $\Rightarrow$ or $\Rightarrow \cup \succ$ or . . .
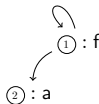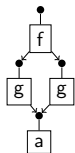
$$(\textcircled{1}, \{\textcircled{1} = f(\textcircled{1}, \textcircled{2}), \textcircled{2} = a\})$$

. . .

Termination

- relation to term rewriting
- application: natural language processing
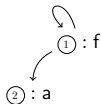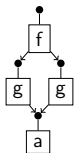- tools GREZ and GREW

③ Literature

Different Representations and $\Rightarrow$ or $\Rightarrow \cup \succ$ or . . .

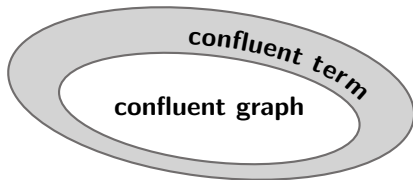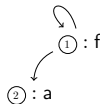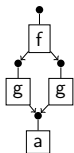Termination **Confluence**

- relation to term rewriting

③ Literature

Different Representations and $\Rightarrow$ or $\Rightarrow \cup \succ$ or ...

$(\text{①}, \{\text{①} = f(\text{①}, \text{②}), \text{②} = a\})$

...

Termination   Confluence   **Modularity**

- relation to term rewriting
- ...subtle but severe differences!
- e.g., confluence not preserved under signature extension

③ Literature

Different Representations and $\Rightarrow$ or $\Rightarrow \cup \succ$ or . . .
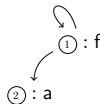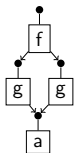
$$(\textcircled{1}, \{\textcircled{1} = f(\textcircled{1}, \textcircled{2}), \textcircled{2} = a\})$$

Termination   Confluence   Modularity   **Sharing & Memoisation**

- not the same, although sharing may save computation
- approach which also incorporates memoisation

Conclusion

Summary

① Termination.

② Collapsing.   e.g. $\Rightarrow \cdot \succsim$ vs $\Rightarrow \cup \succ$

③ Literature.

Conclusion

Summary

① Termination.

 $>_{\mathsf{lpo}}$ $>_{\mathsf{lpo}}$

② Collapsing.   e.g. $\Rightarrow \cdot \succcurlyeq$ vs $\Rightarrow \cup \succ$

③ Literature.

Impact

📄 G Moser, M A Schett. Kruskal's Tree Theorem for Acyclic Term Graphs *Proc. 9th TERMGRAPH*, 2016.

- ACM student research competition at POPL 2016, Florida won 3rd place graduate category
- presented at OPLSS 2016 in Oregon, and LC&A 2016 in Obergurgl

Conclusion

Summary

(1) Termination. $>_{\mathsf{lpo}}$ $>_{\mathsf{lpo}}$

(2) Collapsing. e.g. $\Rightarrow \cdot \succcurlyeq$ vs $\Rightarrow \cup \succ$

Thank you for your attention!

Impact

📄 G Moser, M A Schett. Kruskal's Tree Theorem for Acyclic Term Graphs *Proc. 9th TERMGRAPH*, 2016.

- ACM student research competition at POPL 2016, Florida won 3rd place graduate category
- presented at OPLSS 2016 in Oregon, and LC&A 2016 in Obergurgl

Conclusion

Summary

1. Termination.

 $>_{\text{lpo}}$ $>_{\text{lpo}}$

2. Collapsing.

e.g. $\Rightarrow \cdot \succcurlyeq$ vs $\Rightarrow \cup \succ$

3. Literature.

Impact

📄 G Moser, M A Schett. Kruskal's Tree Theorem for Acyclic Term Graphs *Proc. 9th TERMGRAPH*, 2016.

- ACM student research competition at POPL 2016, Florida won 3rd place graduate category
- presented at OPLSS 2016 in Oregon, and LC&A 2016 in Obergurgl